THE EFFECT OF A MULTIFACETED VARIABLE APPROACH
ON THE LEARNING AND TEACHING OF ALGEBRA
IN THE JUNIOR SECONDARY SCHOOL

by

Salma Tahir, B.Sc., B.Ed., M.Sc., M.Phil.

Department of Education,
Faculty of Human Sciences
Macquarie University

This thesis is presented for the degree of Doctor of Philosophy

October, 2011
# TABLE OF CONTENTS

TABLE OF CONTENTS i

LIST OF APPENDICES vii

LIST OF TABLES viii

LIST OF FIGURES x

DECLARATION xi

ACKNOWLEDGEMENT xii

ABSTRACT xiv

CHAPTER 1  Introduction 1

1.1  Teaching algebra 2

1.2  A proposed new approach 3

1.3  Aim of the study 4

1.4  Significance of the study 4

1.5  Outline of the thesis 5

CHAPTER 2  Student Difficulties in Algebra 6

2.1  Introduction 6

2.2  Difficulties in transition from arithmetic to algebra 6

2.3  Misconceptions regarding the concept of a variable 8

2.4  Problems related to the representation and solution of a linear equation 11

2.4.1  Errors in representation 12

2.4.2  Errors in simplification of algebraic expressions 13
2.4.3 Problems with the interpretation of equality and Equivalence 14
2.4.4 Moving from arithmetic to algebraic solution strategies for linear equations 15

2.5 Issues of engagement 18
2.6 Summary 18

CHAPTER 3 Teaching Approaches 20
3.1 Teaching approaches 20

3.1.1 Problem-solving and modelling approaches 20
3.1.2 Generalisation approaches 24
3.1.3 Functional approaches 26

3.2 Essential elements of a successful algebra program 28

3.2.1 Developing the concept of a variable 29
3.2.2 Promoting algebraic reasoning 30
3.2.3 Breadth and integration 31
3.2.4 Making algebra meaningful and interesting 31

3.3 The Multifaceted Variable Approach 32

3.3.1 The MVA in the present study 35

3.3.1.1 Patterns 36
3.3.1.2 Algebraic techniques 36
3.3.1.3 Linear relationships and coordinate geometry 36

3.4 Summary 37
CHAPTER 4  Methodology  39

4.1  Introduction  39

4.2  Sample  39

4.3  Phase I  41
   4.3.1  Professional Development Workshop I  41
   4.3.2  The algebra teaching program  44
   4.3.3  Lesson observations  45
   4.3.4  First and Second Algebra Tests and interviews  46

4.4  Phase II  47
   4.4.1  Professional Development Workshop II  49
   4.4.2  Algebra teaching  49
   4.4.3  Lesson observations  50
   4.4.4  Third Algebra Test and interview  51

4.5  Justification of methodology  52

4.6  Summary  57

CHAPTER 5  Results: The Teachers  58

5.1  Introduction  58

5.2  Professional Development Workshop I  58
   5.2.1  Teachers’ beliefs about mathematics teaching and learning  58
      5.2.1.1  Rosa  59
      5.2.1.2  Mona  59
5.2.1.3 Amy 60
5.2.1.4 Ben 60
5.2.2 Teachers beliefs about algebra and teaching algebra 61
5.2.2.1 Rosa 61
5.2.2.2 Mona 62
5.2.3 Common student errors and usual sequence of algebra lessons 63
5.2.4 Workshop evaluation 64

5.3 Lesson observations 64
5.3.1 Rosa 65
5.3.2 Mona 68
5.3.3 Amy 69
5.3.4 Ben 71

5.4 Professional Development Workshop II 73
5.5 Summary 74

CHAPTER 6 Results: The Students (Phase I) 77
6.1 Introduction 77
6.2 Learning (Phase I) 77
6.2.1 First Algebra Test 78
6.2.2 Student Interview I 85
6.2.3 Yearly Examination 89
6.2.4 Second Algebra Test 90
6.3 Summary 95
# LIST OF APPENDICES

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix A</td>
<td>Principals’ Information and Consent Form</td>
<td>166</td>
</tr>
<tr>
<td>Appendix B</td>
<td>Teachers’ Information and Consent Form</td>
<td>168</td>
</tr>
<tr>
<td>Appendix C</td>
<td>Parents’ Information and Consent Form¹</td>
<td>170</td>
</tr>
<tr>
<td>Appendix D</td>
<td>Mathematics Teaching Questionnaire</td>
<td>172</td>
</tr>
<tr>
<td>Appendix E</td>
<td>Algebra Questionnaire (I)</td>
<td>174</td>
</tr>
<tr>
<td>Appendix F</td>
<td>Algebra Questionnaire (II)</td>
<td>176</td>
</tr>
<tr>
<td>Appendix G</td>
<td>Workshop Evaluation Form</td>
<td>180</td>
</tr>
<tr>
<td>Appendix H</td>
<td>Examples of Activities from the Resource Book</td>
<td>182</td>
</tr>
<tr>
<td>Appendix I</td>
<td>First Algebra Test</td>
<td>186</td>
</tr>
<tr>
<td>Appendix J</td>
<td>Yearly Examination</td>
<td>192</td>
</tr>
<tr>
<td>Appendix K</td>
<td>Second Algebra Test</td>
<td>194</td>
</tr>
<tr>
<td>Appendix L</td>
<td>Lesson Plan Feedback Sheet</td>
<td>196</td>
</tr>
<tr>
<td>Appendix M</td>
<td>Student Interview Schedule I</td>
<td>198</td>
</tr>
<tr>
<td>Appendix N</td>
<td>Third Algebra Test</td>
<td>200</td>
</tr>
<tr>
<td>Appendix O</td>
<td>Half Yearly Examination (Algebra Problems)</td>
<td>206</td>
</tr>
<tr>
<td>Appendix P</td>
<td>Fourth Algebra Test</td>
<td>210</td>
</tr>
<tr>
<td>Appendix Q</td>
<td>Student Interview II</td>
<td>212</td>
</tr>
<tr>
<td>Appendix R</td>
<td>Ethics Approval Letter</td>
<td>214</td>
</tr>
</tbody>
</table>
LIST OF TABLES

CHAPTER 2

Table 2.1 Algebraic levels, Piaget’s stage of cognitive development, sample items, student understanding of variables 11

CHAPTER 4

Table 4.1 The participating teachers 41
Table 4.2 Student and teacher activities and data collected in Phase I 42
Table 4.3 Student-teacher activities and data collection in Phase II 48

CHAPTER 6

Table 6.1 Mean marks (as percentage) in number, angles and algebra 78
Table 6.2 Mean marks (as percentage) in algebraic skills for first algebra test 79
Table 6.3 Percentage of students who made errors in Question 2 and Question 4 by category 80
Table 6.4 Categories of student responses 86
Table 6.5 Mean interview scores 86
Table 6.6 Mean marks (as percentage) in algebraic skills for algebra problems in the yearly examination 89
Table 6.7 Mean marks (as percentage) in algebraic skills for Second Algebra Test 91
Table 6.8 Percentage of conjoining errors in Question 2 (part b and part c) 92
CHAPTER 7

Table 7.1  Mean marks (as percentage) in algebraic skills for Third Algebra Test  

Table 7.2  Percentage of students making various errors in the Third Algebra Test  

Table 7.3  Percentage of correct responses in algebraic skills on the Half Yearly Examination  

Table 7.4  Percentage of students who made simplification errors in the Half-Yearly Examination  

Table 7.5  Percentage of correct responses of students of each class in solving linear equations  

Table 7.6  The number of students who used the solution strategy and the number of correct responses  

Table 7.7  The mean (as percentage) of students who used the solution strategies  

Table 7.8  Percentage of correct responses by solution strategy
LIST OF FIGURES

CHAPTER 2
Figure 2.1  Working Backwards  

CHAPTER 5
Figure 5.1  Graphs drawn on the white board (a) represents climbing one stair at a time, (b) represents climbing one stair then two stairs, then repeating this pattern  

x
STATEMENT OF CANDIDATE

I hereby certify that the research presented in the thesis entitled “The effect of a Multifaceted Variable Approach on the learning and teaching of algebra in the junior secondary school” is my original work and it has not been submitted for a higher degree to any other university or institution other than Macquarie University.

I also certify that the thesis is an original piece of research and it has been written by me. Any help and assistance that I have received in my research work and the preparation of the thesis itself have been appropriately acknowledged.

In addition, I certify that all information sources and literature used are indicated in the thesis.

The research presented in this thesis was approved by the Macquarie University Ethics Review Committee, protocol number HE22FEB2008-D05638 (February 2008).

Salma Tahir
Student ID: 41102479
30th September, 2011
Acknowledgement

In completion of this study I have benefited from the encouragement and support of two institutions, my supervisors, professional friends and my family.

I am grateful to the Department of Education, Macquarie University for providing me with additional financial support and a learning environment conducive for research and professional development.

I am indebted to AusAID who assisted me by providing me with a scholarship to complete the Doctoral program.

I am grateful to both of my supervisors Dr Michael Cavanagh and Dr Mike Mitchelmore whose visionary approach and professional expertise guided me at every stage of this study.

Michael’s encouragement and support facilitated me right from planning to the final stage of this study. He always guided me with enthusiasm, dedication and professionalism.

Mike’s advice and feedback was invaluable. His expertise in finding patterns and relationships was enlightening to say the least, when the data were analysed.

My special thanks to Dr John Hedberg, Head of Department of Education, Macquarie University who provided moral encouragement and support when it was needed during the course of this project.

My special thanks to the staff and students of the school who participated in the study and welcomed me into their community.

I am extremely grateful for the invaluable discussions, comments and feedback provided by my colleague Debbie Bautista who was always there for me in time of need and also for providing me with technical support to restore my endnote file and in making copies of my lesson recordings.
I am thankful to Dr Heather McMaster who edited this thesis.

I also thank Dr Joanne Mulligan who encouraged and supported me and all the staff of Department of Education, in particular to Alison and Marissa for their facilitation in providing me with logistic support.

On a personal level I would like to thank my husband Tahir Iqbal whose continuous support and encouragement made it possible for me to complete this study.
Abstract

Research has shown that students find algebra a difficult subject. One reason may be that they have a limited concept of a variable. To encourage the development of a deeper understanding of variables, a new teaching approach called the Multifaceted Variable Approach (MVA) was designed. The emphasis in the MVA is on understanding relationships between variables using real contexts and their tabular, algebraic and graphical representations. Students experience several facets of the variable concept in parallel: as unknown numbers, generalised numbers and functions. The MVA approach was used to teach algebra in Stage 4 of the syllabus of the NSW Board of Studies. The students were in Years 7 and 8.

The effect of the MVA on students’ understanding of variables and their algebraic competence was investigated in a two-year case study in a girls’ high school in Sydney. The sample consisted of four streamed classes, called (in decreasing order of ability) Sets 1 to 4, and their teachers. The experimental group (49 students in Sets 2 and 4) was taught using the MVA and the comparison group (54 students in Sets 1 & 3) was taught using a traditional teaching approach. Data were collected from student assessments on six separate occasions and from two rounds of student interviews.

Qualitative and quantitative analysis of the data indicated that, on average, students who were taught via the MVA acquired a deeper concept of “variable” and greater algebraic competency than the comparison group. On average, they demonstrated fewer misconceptions and were more successful in interpreting algebraic expressions, representing word problems in an algebraic form, solving linear equations, and in general algebraic reasoning.

The study also confirmed the strong influence of teachers’ concepts of algebra on the quality of student learning. More sustained teacher professional development may be necessary to ensure the change in teaching practice that is necessary to implement the MVA effectively.