Stochastic Heisenberg Limit: Optimal Estimation of a Fluctuating Phase

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The ultimate limits to estimating a fluctuating phase imposed on an optical beam can be found using the recently derived continuous quantum Cramér-Rao bound. For Gaussian stationary statistics, and a phase spectrum scaling asymptotically as $\omega^{-p}$ with $p > 1$, the minimum mean-square error in any (single-time) phase estimate scales as $\mathcal{N}^{-2(p-1)/(p+1)}$, where $\mathcal{N}$ is the photon flux. This gives the usual Heisenberg limit for a constant phase (as the limit $p \to \infty$) and provides a stochastic Heisenberg limit for fluctuating phases. For $p = 2$ (Brownian motion), this limit can be attained by phase tracking.

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Estimating the phase imposed on an optical beam, by nature or by an agent, is a key task in metrology and communication, respectively. One case of broad relevance is that where the phase varies stochastically in time over a wide range [1–8]. It is only very recently that it has been possible to experimentally demonstrate the quantum enhancement (by a constant factor) of the estimation of such a strongly fluctuating phase, using nonclassical (squeezed) light [9] and homodyne detection with adaptive phase tracking [1,7].

Adaptive phase tracking is a sophisticated measurement technique whereby the phase of the local oscillator (necessary for homodyne detection) is continuously changed in time to follow an estimate of the true phase [1–7]. This enables the phase quadrature of the beam to be monitored at all times, to a good approximation, maximizing the phase information obtained. Previously it has been calculated that phase tracking with squeezed light would enable an imposed phase to be estimated with a mean-square error (MSE) scaling as $\mathcal{N}^{-2/3}$ [2–4]. In contrast, for coherent states (no squeezing) only a $\mathcal{N}^{-1/2}$ scaling can be achieved [2–4]. Here, $\mathcal{N}$ is the mean flux (photons per second) in the beam, and the imposed phase is modeled by Brownian motion.

While experiments in optical phase tracking have not yet demonstrated an improvement over the coherent state scaling of $\mathcal{N}^{-1/2}$, the possibility of doing so in the near future raises pressing theoretical questions: is the MSE scaling of $\mathcal{N}^{-2/3}$, derived assuming adaptive estimation [2–4], the best possible? If not, what is the ultimate limit to estimating a fluctuating phase and how can it be achieved?

For measurement of a constant phase, the fundamental bound is the Heisenberg limit [10,11]: a phase estimate MSE scaling as $\langle N \rangle^{-2}$, where $\langle N \rangle$ is the mean number of photons per estimate. This a quadratic improvement over the $\langle N \rangle^{-1}$ scaling achievable using coherent states [the standard quantum limit (SQL)] [10,11]. Hence, if quantum mechanics similarly allowed a quadratic improvement in the case of a fluctuating phase, the corresponding fundamental limit for the MSE would scale as $\mathcal{N}^{-1}$.

Contrary to this intuition, we prove in this Letter, with only weak assumptions, that the fundamental bound to estimating Brownian phase fluctuations is a MSE scaling as $\mathcal{N}^{-2/3}$. This establishes that adaptive phase tracking can be a very effective measurement technique for this problem, giving an uncertainty at most a constant factor greater than the minimum allowed by quantum mechanics (under our assumptions). This $\mathcal{N}^{-2/3}$ scaling for Brownian fluctuations is just a special case of our general stochastic Heisenberg limit, which allows for any inverse power law describing the phase fluctuation spectrum at high frequencies, and which also yields the constant-phase Heisenberg limit as a special case.

This Letter is organized as follows. First, we derive the general stochastic Heisenberg limit and the stochastic SQL. Next we specialize to the scenario of Ref. [1]: a squeezed beam comprising the output of an optical parametric oscillator (OPO) with an added mean field, and a phase varying like damped Brownian motion. We consider the ultimate limit, and find the same scaling as in the general case, but with an explicit constant of proportionality, consistent with the numerics of Ref. [4].

General proof.—Our result applies to the situation of a continuous beam [a one-dimensional quantum field $b(t)$], on which there is an imposed phase $\varphi(t)$. We require only three conditions. (1) The statistics of the field quadratures and imposed phase are stationary. (2) The statistics of the field quadratures and imposed phase are Gaussian and time symmetric. (3) The phase spectrum scales as $|\omega|^{-p}$ for large $|\omega|$, for some $p > 1$.

We now explain these conditions in more detail. The instantaneous creation operator $b^\dagger(t)$ of the beam obeys $[b(t), b^\dagger(t')] = \delta(t - t')$, and $[b^\dagger(t), b(t)] = \mathcal{N}$ is the photon flux [11]. The stationarity of its statistics, and those of $\varphi(t)$, means that one-time expectation values are constant and two-time correlations depend only on the time difference.
The assumed Gaussian statistics mean there is a Gaussian Wigner functional for the quadratures \[ 11 \]

\[ X(t) := b(t)^2 + b(t), \quad Y(t) := [b(t)^2 - b(t)], \tag{1} \]

and a Gaussian distribution for the phase \( \varphi(t) \). For a single Gaussian variable, such as \( \varphi(t) \), the autocorrelation function is automatically time symmetric. However, the beam quadratures \( X(t) \) and \( Y(t) \) may be correlated, and our derivation below requires that this cross correlation be time symmetric (i.e., invariant under \( t \to t' \)).

The third condition allows for a vast range of phase fluctuation models. For \( p = 2 \), it means that at short times the fluctuations are like Wiener noise \[ 11 \] (Brownian motion), which has the spectrum \( \kappa/\omega^2 \), where \( \kappa \) is a constant with units of frequency. More generally, we take the scaling constant to be \( \kappa^{p-1} \), so \( \kappa \) still has units of frequency. In the limit \( p \to \infty \), the phase is effectively constant.

In Ref. \[ 12 \], a continuous form of the quantum Cramér-Rao inequality was derived, giving a lower bound on the MSE of any unbiased estimate \( \hat{\varphi}(t) \) of a time-varying parameter \( \varphi(t) \),

\[ \langle (\hat{\varphi}(t) - \varphi(t))^2 \rangle \geq F^{-1}(t,t). \tag{2} \]

Here, \( F(t,t') \) is the Fisher information matrix (with continuous indices \( t \) and \( t' \)) of the phase of the beam, given by a sum of quantum and classical contributions

\[ F(t,t') := F^{(Q)}(t,t') + F^{(C)}(t,t'), \tag{3} \]

and the (matrix) inverse in Eq. (2) is defined by

\[ \int ds F^{-1}(t,s) F(s,t') = \delta(t-t'). \tag{4} \]

For the case where \( \varphi(t) \) is an imposed phase,

\[ F^{(Q)}(t,t') := 4(\Delta n(t)\Delta n(t')), \tag{5} \]

\[ F^{(C)}(t,t') := \int D\varphi P[\varphi] \delta \ln P[\varphi] \delta \ln P[\varphi] \frac{\delta \varphi(t)}{\varphi(t)} \frac{\delta \varphi(t')}{\varphi(t')}. \tag{6} \]

In the above, \( \int D\varphi \cdot \cdot \cdot \) denotes an integral over all possible functions \( \varphi(t) \), the functional \( P[\varphi] \) gives the prior weight for each function, and \( \delta/\delta \varphi(t) \) is a functional derivative. Also, \( \Delta n(t) = n(t) - \langle n(t) \rangle \), where \( n(t) := b(t)^2 + b(t) \) is the generator of the phase shifts, the photon flux operator. Because of the stationarity condition, all quantities depend on two times \( t \) and \( t' \) are functions only of \( t-t' \). We will express these quantities explicitly as functions of \( t-t' \) from here on. In particular, the lower bound in Eq. (2) will be denoted by \( F^{-1}(0) \).

To determine \( F^{-1}(0) \), substitute Eq. (3) into Eq. (4) and take the Fourier transform, to give

\[ \tilde{F}^{-1}(\omega) = \frac{1}{F^{(C)}(\omega) + F^{(Q)}(\omega) - F^{(Q)}(0)}. \tag{7} \]

for the Fourier transform of \( F^{-1}(t-t') \). The value of \( F^{-1}(0) \) is then obtained by integrating this over \( \omega \). Our aim is to determine the minimum possible scaling of this value with the photon flux \( \mathcal{N} = \langle n(t) \rangle \).

As we assume the phase fluctuations are Gaussian, we have \( F^{(C)}(t-t') = \Sigma(t-t')^{-1} \), with \( \Sigma(t-t') := \langle \varphi(t)\varphi(t') \rangle - \langle \varphi \rangle^2 \) \[ 13 \]. For the case \( \Sigma(\omega) = \kappa^{p-1}/|\omega|^p \),

\[ \tilde{F}^{-1}(\omega) = \frac{\kappa^{p-1}}{|\omega|^p + \kappa^{p-1}\tilde{F}^{(Q)}(\omega)}. \tag{8} \]

More generally, we can obtain the result below with the weaker requirement that the phase spectrum approaches this scaling at high frequencies, i.e., \( \tilde{\Sigma}(\omega) = \Omega(\kappa^{p-1}/|\omega|^p) \) as per condition 3 (see Sec. D of the Supplemental Material \[ 14 \]).

Next we consider the quantity \( F^{(Q)}(t-t') \). This may be simplified to (see Sec. A of the Supplemental Material \[ 14 \])

\[ F^{(Q)}(t-t') = 4\mathcal{N}\delta(t-t') + f(t-t') - g(t-t'), \tag{9} \]

where, in terms of the quadrature operators (1),

\[ f(t-t') := \frac{1}{2} \langle X(t)X(t') \rangle + \langle Y(t)Y(t') \rangle, \tag{10} \]

\[ g(t-t') := \langle X(t)X(t') \rangle; \langle Y(t)Y(t') \rangle - \frac{1}{2} \langle X(t)Y(t') \rangle^2 \]

\[ + \langle Y(t)X(t') \rangle^2 + \frac{1}{2} \langle X(t)^2 + Y(t)^2 \rangle. \tag{11} \]

Thus, from Eq. (8), we obtain

\[ \tilde{F}^{-1}(\omega) = \frac{\kappa^{p-1}}{|\omega|^p + \kappa^{p-1}[4\mathcal{N} + f(\omega) - \tilde{g}(\omega)]}. \tag{12} \]

The photon flux can be written as

\[ \mathcal{N} = \frac{1}{4} (\langle X(t)X(t') \rangle + \langle Y(t)Y(t') \rangle), \tag{13} \]

and therefore \( f(0) = 8\mathcal{N}^2 \). In addition, using a spectral uncertainty principle and the assumption of time-symmetric correlations, it can be shown that \( \mathcal{N}/4 \geq -\tilde{g}(\omega) \) and \( \tilde{f}(\omega) \geq 0 \) (see Sec. B of the Supplemental Material \[ 14 \]). Since it is easily shown that \( \tilde{F}^{(Q)}(t-t') \) is a positive-definite function, by Bochner’s theorem \( \tilde{F}^{(Q)}(\omega) \geq 0 \) \[ 15 \], and thus the denominator in Eq. (12) is positive. Consequently, replacing \(-\tilde{g}(\omega) \) with \( \mathcal{N}/4 \) can only decrease the right-hand side, that is, with \( \zeta = 17/4 \),

\[ \tilde{F}^{-1}(\omega) \geq \frac{\kappa^{p-1}}{|\omega|^p + \kappa^{p-1}[\zeta\mathcal{N} + \tilde{f}(\omega)]}. \tag{14} \]

Next, from the fact that \( \tilde{f}(0) = 8\mathcal{N}^2 \), the integral of \( \tilde{f}(\omega) \) is \( I = 16\pi \mathcal{N}^2 \). This means that \( \tilde{f}(\omega) \) cannot be larger than \( \mu \) over a range greater than \( I/\mu \). To place a lower bound on \( F^{-1}(0) \), when integrating Eq. (14) we may first omit the range of integration where \( \tilde{f}(\omega) \approx \mu \), and replace \( \tilde{f}(\omega) \) by \( \mu \) over the remaining portion. Second, we
can assume that the range of integration omitted is for the smallest values of $\omega$ because that can only further reduce the value of the integral. Since this range can be at most $I/\mu$ in length, this yields
\[
F^{-1}(0) \geq \frac{1}{\pi} \int_{I/\mu}^{\infty} \frac{\kappa^{p-1}d\omega}{\omega^p + \kappa^{p-1}(\zeta N + \mu)}
\geq \frac{1}{2\pi} \int_{0}^{\infty} \frac{\kappa^{p-1}d\omega}{\kappa^{p-1}(\zeta N + \mu)}
\geq \frac{2}{\pi[I/\mu]^{p-1}(\zeta N + \mu)]^{-1/p}},
\]
where the inequality on the second line holds for $(I/\mu)^p \leq \kappa^{p-1}(\zeta N + \mu)$ (see Sec. C of the Supplemental Material [14]).

To obtain the strongest lower bound on $F^{-1}(0)$, we consider the smallest value of $\mu$ that we can take such that $(I/\mu)^p \leq \kappa^{p-1}(\zeta N + \mu)$. This value is $\mu = \Theta(N(N/k)^{(p-1)/p})$. We consider scaling for large $N/k$, in which case $\mu \gg N$, and $\zeta N$ can be ignored. Equations (2) and (15) thus yield our main result, the lower bound scaling for the MSE,
\[
\langle \tilde{\phi}(t) - \phi(t) \rangle^2 = \Omega((\kappa/N)^{(2(p-1)/(p+1)})). \tag{16}
\]
Note that this scaling cannot be achieved by a coherent-state beam, for which $F^{(0)}(t-t') = 4N \delta(t-t')$. It is easy to show, for this case, by taking $f = g = 0$ in Eq. (12), that
\[
\langle \tilde{\phi}(t) - \phi(t) \rangle^2_{\text{SQL}} = \Omega((N/k)^{(p-1)/p}), \tag{17}
\]
which we call the stochastic SQL scaling.

In the case of Wiener phase fluctuations ($p = 2$), the stochastic Heisenberg scaling is $(\kappa/N)^{2/3}$. A simplified analysis in Ref. [2] found that adaptive homodyne measurements can yield this scaling, but it did not take into account either the photon flux due to the squeezing or the information in the photocurrent noise. A more complete analysis, taking both of these terms into account, was performed in Ref. [4] (correcting an error in the analysis of Ref. [3]). That analysis verified the scaling of $(\kappa/N)^{2/3}$ for the MSE. That is, the lower bound in Eq. (16) is attainable by adaptive measurements for $p = 2$.

The limit $p \to \infty$ gives a very slowly varying phase. In that case Eq. (16) gives the expected constant-phase Heisenberg limit scaling $\langle \tilde{\phi}(t) - \phi(t) \rangle^2 = \Omega((N)^{-2})$. Similarly, Eq. (17) gives the expected SQL scaling $\langle \tilde{\phi}(t) - \phi(t) \rangle^2_{\text{SQL}} = \Omega((N)^{-1})$. For all other $p$, the quantum enhancement is less than quadratic, and as $p \to 1$ there is no quantum advantage.

**OPO squeezing and Ornstein-Uhlenbeck fluctuations.**—Next we specialize to the model of squeezing used in Refs. [1–3]—a coherent field of real amplitude $\alpha$ added to an OPO output—and to phase fluctuations modeled by Ornstein-Uhlenbeck noise, with $\Sigma(\omega) = \kappa/\lambda^2 + \omega^2$ as in Ref. [1]. Asymptotically, this is identical to the Wiener phase spectrum ($p = 2$) analyzed above. For this beam we have $\langle X \rangle = 2\alpha$, $\langle Y \rangle = 0$, so Eq. (9) becomes
\[
F^{(0)}(t-t') = 4N \delta(t-t') + 4\alpha^2 T_+(t-t')
\]
\[
+ \{[T_+(t-t')]^2 + [T_-(t-t')]^2\}/2, \tag{18}
\]
where $T_\pm(t-t')$ are the normally ordered correlation functions for the quadrature fluctuations [1,16]
\[
T_\pm(t-t') = (\Delta Q_\pm(t) \Delta Q_\pm(t'))/2, \tag{19}
\]
where $Q_+ := X$ and $Q_- := Y$. They are given by
\[
T_\pm(t-t') = (R_\pm - 1)(1 + x) \gamma e^{-1(1 \mp \gamma)|t-t'|/2}. \tag{20}
\]
In Eq. (20), $R_\pm$ are the antisqueezing and squeezing levels, respectively, at the center frequency. For an OPO, $\gamma$ is the cavity’s decay rate [17] and $x \in [0, 1]$ is the normalized pump amplitude. In terms of these quantities, the total photon flux is
\[
N = \alpha^2 + \gamma^2 /16 [(R_+ - 1)(1-x) + (R_- - 1)(1+x)]. \tag{21}
\]
Substituting these expressions in Eq. (18) and taking the Fourier transform yields
\[
F^{(0)}(\omega) = 4N + 4\alpha^2 (R_+ - 1)(1-x)^2 \gamma^2 /16 [1 - (1-x)^2 \gamma^2 + 4\omega^2]
\]
\[
+ \gamma^2 [R_+ - 1(1-x)^3 /16 (1-x)^2 \gamma^2 + \omega^2 + (R_- - 1)(1+x)^3 + (1+x)^2 \gamma^2 + \omega^2] \tag{22}
\]
For a coherent state ($R_+ = R_- = 1$), we would just have $F^{(0)}(\omega) = 4N$, and we would obtain $\kappa/(2\sqrt{4N\kappa + \lambda^2})$ as the lower bound on the MSE. This coherent state limit was first derived by Tsang et al. [see Eq. (4.5) in Ref. [5]] and scales asymptotically as $(\kappa/N)^{1/2}$ as expected [2].

**Comparison with the experiment of Ref. [1].**—It seems impossible to obtain an exact analytical solution for $F^{-1}(0)$ in the case of general OPO squeezing. However, a useful approximation is to just include the terms in Eq. (22) that represent information available from the mean field, that is, those terms proportional to $\alpha^2$. As in the theory of Ref. [2], the estimation performed in Ref. [1] used only the signal from the mean field, so this approximation is relevant to those works. As in those works we also express our results in terms of $\alpha^2$, rather than $N$. Then we find
\[
F^{-1}(0) = (\kappa/2)(1 + g^2 \sqrt{\Xi_+ / \Xi_-}), \tag{23}
\]
where
\[
\Xi_\pm = \frac{1}{2} [4\alpha^2 \kappa + \lambda^2 + g^2 \pm \sqrt{(4\alpha^2 \kappa + \lambda^2 - g^2)^2 - 4d}], \tag{24}
\]
with $g = (1-x)\gamma/2$ and $d = 4\kappa \alpha^2 (R_+ - 1)g^2$. 

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With further simplification this bound on the MSE is comparable to the results given for adaptive measurements on squeezed states in Ref. [1]. First we note that a mixed squeezed state described by $R_+, \gamma$ and $x$ can be assumed to be a combination of a pure squeezed state and classical amplitude noise (see Sec. E of the Supplemental Material [14]), where the pure squeezed state is described by $R_0 = R_-, \quad \gamma = 1 / R_+ \quad x = (\sqrt{R_+} - 1) / (\sqrt{R_+} + 1)$, and $\gamma^2 = (1 + x) / (1 + x^2).$ Then one can determine $F^{(0)}(t)$ from the parameters for the pure squeezed state.

Using this approach, in the limit of large bandwidth, $\gamma \rightarrow \infty$, we obtain $\kappa / (2 \sqrt{4 \alpha^2 R_+ \kappa + \lambda^2})$ as the lower bound on the MSE. This expression is that shown as trace (iii) in Fig. 3 of Ref. [1], derived from Eq. (3) of Ref. [1] by taking $\sigma_f \rightarrow 0$ (the limit of perfectly accurate feedback). Note that this is significantly below what was observed in the experiment, because the mean-field adaptive algorithm used in the experiment was far from being perfectly accurate.

**Ultimate limit for OPO squeezing and $p = 2$.**—We have already shown that the lower bound to the MSE for $p = 2$ scales as $(\kappa / \mathcal{N})^{2/3}$. Now we show how to determine the constant of the scaling for this model assuming ideal OPO squeezing with $R_+ = 1 / R_$. We include all terms in Eq. (22) and express our results in terms of $\mathcal{N}$. We introduce dimensionless (starred) parameters via $\mathcal{N} = \kappa \mathcal{N}_*, \quad \alpha^2 = \alpha_+^2 \kappa \mathcal{N}_*, \quad \gamma = \gamma_+ \kappa \mathcal{N}_x^{5/6}, \quad R_+ = R_+ \mathcal{N}_x^{1/3}, \quad \omega = \omega_+ \mathcal{N}_x^{1/3}$, where $\gamma_+ > 0$ and $R_+ > 0$. Then we obtain in the limit $\mathcal{N}_x \rightarrow \infty$ (see Sec. F of the Supplemental Material [14])

$$F^{-1}(0) = \mathcal{N}_x^{-2/3} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_+}{\omega_+^2 + \tilde{F}^{(0)}(\omega_+)} ,$$

(25)

with

$$\tilde{F}^{(0)}(\omega_+) = \frac{4 \gamma_+^2 \alpha_+^2}{\gamma_+^2 / R_+ + \omega_+^2} + \frac{\gamma_+^3 \sqrt{R_+}}{4 \gamma_+^3 / R_+ + \omega_+^2} .$$

(26)

From Eq. (21), the above dimensionless parameters are related by $\alpha_+^2 = 1 - \gamma_+ \sqrt{R_+} / 8$. It is convenient to define $\tau = \gamma_+ \sqrt{R_+} / 8$, so the allowable values for $\tau$ range from 0 to 1. The value of $C := \mathcal{N}_x^{2/3} F^{-1}(0)$ was calculated for this range of $\tau$, and $\gamma_x \in [0, 4]$; the results are given in Fig. 1. It can be seen that $C$ is smallest for $\tau = 1$, which corresponds to a squeezed vacuum, and the minimum is $C_0 = (587 - 143 \sqrt{13}) / (4 \sqrt{6}) = 0.2078$ for $\gamma_x = 2 (2 \sqrt{13} - 3)^{1/6} = 1.7319$. That is,

$$\langle [\hat{\varphi}(t) - \varphi(t)]^2 \rangle \geq C_0 (\kappa / \mathcal{N})^{2/3} ,$$

(27)

with $C_0 = 0.2078$. In this limit of a squeezed vacuum it is only possible to obtain the estimate of the phase modulo $\pi$. However, the shallowness of the plot with $\tau$ shows that one can obtain close to the optimal value for large coherent amplitude, so the phase can be measured modulo $2 \pi$. The phase tracking simulations in Ref. [4] showed that it is possible to estimate $\varphi$ modulo $2 \pi$ with $\langle [\hat{\varphi}(t) - \varphi(t)]^2 \rangle \approx (\kappa^2 / \mathcal{N}^2)^{1/3}$. Moreover, those simulations obtained $\hat{\varphi}(t)$ by filtering the data prior to $t$. By using smoothing [18] of the data before and after $t$, one would halve this MSE [1,5–7].

**Conclusion.**—In summary, we have found a stochastic form of the Heisenberg limit for measurements of a fluctuating phase imposed on a beam with time-invariant statistics. For Wiener fluctuations, the scaling of $(\kappa / \mathcal{N})^{2/3}$ is tight, in that there is a known adaptive measurement scheme that achieves it. Our bound also reproduces the (tight) Heisenberg scaling of $(\kappa / \mathcal{N})^{-1}$ for an effectively constant phase. We thus conjecture our general bound to be tight for all power-law phase spectra. We do note, however, that we have assumed a beam with time-symmetric Gaussian statistics, and it is an interesting open question to prove (or disprove) our bound without this assumption.

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[16] In Ref. [1], the correlations, denoted $X_{\pm}(\tau)$, are symmetrically ordered so that $X_{\pm}(\tau) = T_{\pm}(\tau) + \delta(\tau)$.
[17] In Ref. [1], $2\Delta \Omega_0$ is used instead of $\gamma$.
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