Analysis of Chaotic Semiconductor Laser Diodes

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Abstract—Developments in measurement equipment bandwidth and memory have allowed standard chaotic time series analysis of the multi-GHz output power fluctuations of a semiconductor laser operating chaotically. An investigation into the effect of noise shows the degradation of this analysis at low signal-to-noise ratios.

Keywords - chaos; semiconductor laser, analysis, dimension

I. INTRODUCTION

It is well known that semiconductor lasers can exhibit instabilities and chaotic dynamics when operated as nonlinear devices and systems [1]. There is currently much interest in chaotic semiconductor lasers (CSL) as the ability to synchronize the output of transmitter and receiver CSL’s has many potential applications for secure optical communication schemes. It has recently been shown that the chaotic output of such lasers can be used to mask signals to be sent through commercial fibre-optic communication networks and successful recovery of the hidden message can be accomplished [2].

As such, it is important to analyze the output power time series of CSL systems and to quantify the chaos observed. There are many well established techniques for characterizing simulated chaotic systems. Reference [3] provides a good summary of many of the techniques. Although each technique provides some insight into the dynamics of chaotic systems, there are some that are more suited for application to experimental data. The biggest obstacle to performing reliable and robust chaos data analysis is that the techniques rely heavily on having a large set of noise free data, sampled at a sufficiently high rate compared to the characteristic time scale of the dynamics. This is trivial for simulated chaotic data for which full control over sampling rate and data set size is possible. The only noise present in simulated data comes from the lack of precision in the numerical calculations. In most cases this has little to no effect on the chaos analysis. This is in stark contrast with experimentally recorded chaos where sampling rate and data set size are limited by the equipment being used and the nature of the chaotic signal itself. Noise contamination in most real world chaotic systems is unavoidable.

Regardless of this, chaos data analysis is commonly undertaken on observed chaotic data. In general, the dynamics observed experimentally and successfully analyzed come from systems with low noise levels and relatively low frequencies, allowing the chaotic time series to be accurately recorded. Past analysis of CSL systems have been unsuccessful due to the fast timescales of the output power fluctuations (sub nanosecond and picosecond) and the resulting incompletely captured time series. Sources of noise arising from the detector, oscilloscope and laser itself also contribute to the failure of analysis techniques on CSL experimental data.

Advancements in fast photodiode detectors and real-time oscilloscopes, which are now available with bandwidths up to 18 GHz, large amounts of memory and sampling rates up to 40 giga-samples per second, now allow time series to be recorded much more precisely and for longer periods of time than ever before. This improved data quality means that it is now valid to revisit chaos data analysis of the output power time series of CSL systems.

II. THEORY

A. Chaotic Semiconductor Laser

A common method employed to obtain chaotically varying output power from a semiconductor laser is to reflect some of the output light back into the active region of the laser. Semiconductor lasers can be described by three rate equations for amplitude, phase and carrier density [4]. It is possible to simulate optical feedback by simply adding a time-delayed feedback term to the standard equations. These equations with the feedback term are known as the Lang-Kobayashi equations [5]. This feedback term has the effect of coupling the amplitude and phase term together, producing a system of three, coupled, nonlinear differential equations, a condition necessary for a system to exhibit nonlinear dynamic instabilities [6].

In practice, the effect that this feedback has on the output of the laser depends on the amount and phase of light that is reflected [7]. For some levels of feedback it is possible to stabilize the laser output into single mode operation, reduce the noise amplitude and narrow the linewidth compared to the solitary laser output. For a certain region of feedback levels the laser enters what is known as ‘coherence collapse’ and it is within this region that a chaotically fluctuating output power may be produced.

The diagram in Fig. 1 shows the local maxima and minima from generated time series with increasing feedback (the feedback parameter is a measure of the strength and phase of the reflected light [4]). The time series were simulated by solving the Lang-Kobayashi equations for semiconductor lasers with optical feedback [5]. The laser output amplitude can be seen to go through a series of bifurcations before developing into full chaos as the feedback level is increased. This is the region of interest for this work.
B. Chaos Analysis

Many chaos analysis techniques require a phase space trajectory, or attractor, to be constructed in order to observe the complex dynamics. Most observations made on real world chaotic systems involve measurements of only one parameter, from which an attractor must be formed. These attractors can be constructed using the embedding method [8], which takes a single scalar measurement \( x(n) \) and creates a set of vectors which describe the motion of the dynamics in a \( d \)-dimensional space. The vectors are of the form,

\[
y(n) = [x(n), x(n+T), x(n+2T), \ldots, x(n+(d-1)T)].
\]

This embedding requires values for the time delay \( T \) and embedding dimension \( d \) to be chosen. There are several methods that are commonly employed to determine these parameters. As a general rule the choice of time delay \( T \) must be a multiple of the sampling time and ensure that data is separated as much as possible without the points becoming completely independent. The embedding dimension \( d \) must be large enough that the orbit of the attractor is completely unfolded so that no overlaps due to projection occur.

One chaos analysis technique that is well suited for application to experimental data is fractal dimensions. There are many different definitions of fractal dimensions, but the most appropriate one to use for experimental data is the correlation (or Grassberger) dimension [9]. The correlation dimension characterizes the shape of a chaotic attractor. A higher correlation dimension indicates the trajectory has a higher level of complexity, arising from a more chaotic system.

The correlation dimension was calculated using the Grassberger-Procaccia algorithm [9]. This involves calculation of the correlation integral defined as

\[
C_m(r) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} H \left( r - \| y_i - y_j \| \right),
\]

where \( H(x) \) is the Heaviside function (\( H = 1 \) if \( x \geq 0 \) and \( H = 0 \) if \( x < 0 \)). The value \( r \) is the radius of a hypersphere in an \( m \)-dimensional space and \( N \) is the number of points in the space.

In theory the density of the points will scale as a power law with the radius

\[
\lim_{N \to \infty} \lim_{r \to 0} C_m(r) = r^D.
\]

The correlation dimension is then given by

\[
D = \lim_{N \to \infty} \lim_{r \to 0} \frac{\log C_m(r)}{\log r}.
\]

So the gradient of a graph of \( \log C_m(r) \) versus \( \log r \) should approach the correlation dimension in the limit as \( r \to 0 \). For experimental data that has finite sampling, the slope of this graph for very small \( r \) is very inaccurate so instead a region of \( r \) is taken over which the slope is relatively stable.

III. METHOD & RESULTS

The laser used as the chaotic source was an index-guided, multiple quantum well STC laser diode (LT50-03U) operating at a wavelength of ~850 nm. Using a GRIN lens, the output was collimated into a beam splitter, from which one beam was sent to a fast photodiode (Alphalas UPD-40-VSI-P) for analysis and the other to an external mirror (\( R = 99\% \)) to provide feedback. The level of feedback was controlled using a continuously variable attenuator. This was adjusted so that the feedback produced a chaotically fluctuating output power. The setup is shown in Fig. 2.

The output power time series was captured on a 4 GHz digital oscilloscope (Agilent Infiniium 86142B) capable of recording 20 giga-samples per second with a memory of 8 MB. A variable attenuator was placed in front of the photodiode to control the signal strength.

A correlation dimension measured from chaotic data will saturate to a fixed level as the data is embedded in higher dimensions. A time series from a purely stochastic process will give rise to a correlation dimension that continues to increase for higher embedding dimensions. This trend is shown in Fig. 3 where the chaos curve is from the chaotic Lorenz equations [10] and the noise curve is from a set of randomly generated numbers.

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In order to check the validity of results a surrogate data set must be created and subject to the same analysis [11]. In this work the surrogate data sets used were constructed using the original time series by taking the Fourier transform, randomizing the phases and calculating the inverse Fourier transform.

It is well known that noise has a major impact on nonlinear analysis techniques. An experimental investigation was carried out in order to assess how the signal-to-noise ratio (SNR) of the laser output power time series impacted on the calculation of correlation dimension. The signal-to-noise ratio is defined as

\[
SNR = \frac{\text{Signal Peak to Peak Amplitude}}{\text{Noise Peak to Peak Amplitude}}
\]  

Since the experimental noise should be constant for each time series measurement, only the signal strength need be varied. This was done by changing the transmission of the variable attenuator in front of the photodiode.

Phase space trajectories were constructed from the recorded time series as described in (1). In this work the time delay was taken as the first minimum of the mutual information function [12] and the embedding dimension was successively increased for each calculation. Each trajectory was subject to an algorithm implementing (2) and the slope of the curves over the middle quarter was taken as the correlation dimension. The software program used for this was Chaos Data Analyzer: Professional Version (Physics Academic Software).

The graphs in Fig. 4 show the correlation dimension versus embedding dimension for varying SNR. It can be seen that for the highest SNR, the graph Fig. 4(a) has a reasonably well defined plateau, indicating that the correlation dimension estimate is remaining fairly constant at higher embedding dimensions. This is what is expected of chaotic dynamics as once the trajectory is embedded in sufficiently high enough dimension to completely unfold it, further embedding should have no impact on the shape, and therefore dimension, of the attractor formed. As the SNR is decreased the graphs in Fig. 4(b) to Fig. 4(d) show the correlation dimension saturating at higher values. Further decreases in the SNR result in graphs that show no evidence of reaching a fixed correlation dimension value, see Fig. 4(e) to Fig. 4(h). This indicates severe noise corruption.

The surrogate data curves reinforce the fact that the time series are chaotic in nature. If a clear distinction between the experimental data and the surrogates can be made then it can be implied that the analysis is valid. The results seen in Fig. 4 also show that the surrogates and the original curves are separated more when the SNR is high. At low SNR the fact that the two curves approach each other means that we cannot be as confident in the correlation dimension estimates.
Working with laser diodes as a source of chaos is a useful tool for studying many of the features of nonlinear systems. The vast majority of chaotic systems provide very little opportunity for systematic investigations. Weather, neural activity and financial fluctuations are all examples of chaotic systems for which very little control over system parameters is available. Semiconductor lasers operating chaotically give us a unique opportunity to study many aspect of chaos with the ability to alter many system parameters. This work in particular presents analysis of chaos with different signal to noise ratios; however there are many other variables that can also be easily modified and their effects studied.

IV. CONCLUSION

These investigations have shown that, with the improved data quality now available from modern measurement equipment, it is possible to perform reliable chaos data analysis on the output power time series from a chaotic semiconductor laser, provided that the signal is strong enough to overcome the degrading effect of the noise.

ACKNOWLEDGMENTS

This research is supported by an Australian Research Council Discovery Project grant and by Macquarie University. J Toomey is supported by an Australian Postgraduate Award PhD scholarship.

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